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Optimization of resource allocation process in networks of ground stations

Optimización de proceso de asignación de recursos en redes de estaciones terrenas

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Abstract
Research into engineering and technology, as in other areas of knowledge, presents common issues when it is required to allocate an insufficient number of resources in the most optimal manner possible; in the case of this research paper this means assigning a greater number of satellites to a lesser number of ground stations, considering the input variables generated by the cost function. The Hungarian optimization algorithm constitutes an initial possible solution that can be applied to various cases. However, its approach and mathematical formulation leads to a non-square matrix of variables; therefore, it requires an adaption of the algorithm called the Adapted Hungarian Algorithm, making use of dummy variables. Thus, the objective of this study is to present an adaptation and formalization of the Hungarian algorithm for the case of non-square matrices, in the framework of the problems in allocating n satellites to m ground stations, with m<n. It is concluded that the formalization of the optimization structure of the Adapted Hungarian Algorithm generates plausible or coherent solutions to a cost function, facilitates its application and implementation to a wide range of situations and it can be executed through easy access software.
Key words: autonomous agent, ground station network, hungarian algorithm.

Resumen
En este artículo, presentamos los resultados del proceso de asignación de recursos en redes de estaciones terrestres para el seguimiento y control de satélites. El sistema autónomo y dinámico para estaciones terrestres (ADSGS por sus siglas en ingles), requiere asignar los recursos de las estaciones terrenas a los satélites que hacen un paso sobre ellos en un tiempo específico, esto último solo ocurre durante un tiempo promedio de 7 a 8 minutos y una estación terrena puede unicamente asistir a un satélite en ese momento, aquí está en problema. ¿Qué hacer con los otros satélites que están pasando en ese momento en la estación terrestre y, sobre todo, a qué satélite asistir? Para el desarrollo de ADSGS, se utilizaron técnicas de inteligencia artificial, una red de estaciones terrestres experimentales SATNet, un agente inteligente que utiliza el algoritmo húngaro para optimizar la asignación de satélites a estaciones terrenas y el software de seguimiento. Para resolver el problema planteado, se sugirió la creación de un agente autónomo para realizar esta tarea y, junto con el algoritmo húngaro, optimizar la asignación de recursos, además, el modelo matemático que conduce a la solución de la propuesta. Finalmente, se desarrollan las ecuaciones que conducen al desarrollo de dicho modelo matemático y se muestran las variables se pueden adaptar y usar para cualquier escenario donde hay n número de satélites y m número de estaciones.
Palabras clave: agente autónomo, algoritmo hungariano, redes de estaciones terrenas.

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1. Introduction

Research in spatial engineering and technology is increasingly complex. This study presents a common problem which arises when allocating the existing software and hardware resources between ground stations and satellites.

There are various considerations for this problem. For example: a ground station can only serve a single satellite at a given moment; there are less ground stations than satellites; each mission of the satellite has a unique configuration regarding its frequency band, modulation scheme, proximity, visibility and priority, among other variables.

The aim of this research work is the optimization of the allocation process of said resources through the Hungarian algorithm, taking into consideration established variables as a basis to design an autonomous agent capable of allocating a satellite to a ground station in a time t.

For its development, different concepts were taken into consideration, such as autonomous agents, ground station networks, satellite tracking and control, band frequencies necessary to define a satellite link, and the Hungarian algorithm created by Harold W. Kuhn in 1955, framed within a mathematical model.

Throughout this article there are conceptual references, the methodology used to develop this research work, the results obtained and, finally the conclusions derived from this investigation.

2. Theoretical Analysis

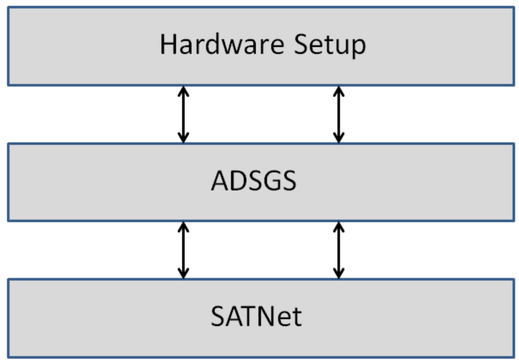
In this section the concepts used for the development of this work are defined: ADSGS, Autonomous Agent, Hungarian Algorithm, similarly, the problem to be solved is formulated and possible situations or scenarios are raised in which they worked.

2.1. ADSGS – Autonomous and dynamic system for ground stations

According to (Espindola, 2017), the Autonomous and Dynamic System for Ground Stations (ADSGS) is the solution for the tracking and control of satellites, which includes a configuration of hardware (antennas, radios, controllers, amplifiers, etc.), and of software necessary for the system to predict orbits and track satellites, carry out satellite links, send telecommands and receive the telemetries of the satellite. ADSGS uses a System Based Ruler (SBR) as an artificial intelligence technique, innovates in the way of implementing Software Defined Radio (SDR), based on an experimental network of SATNet ground stations to complete the autonomous and dynamic systems. The proposal developed in 2017 by (Espindola, 2017) was sponsored by the Universidad Pedagógica y Tecnológica de Colombia through the research INFELCOM and GAMMA, as part of a doctoral thesis.

ADSGS then becomes a middleware between SATNet and the hardware configuration of the ground station. Figure 1 shows this architecture.

Figure 1
ADSGS architecture



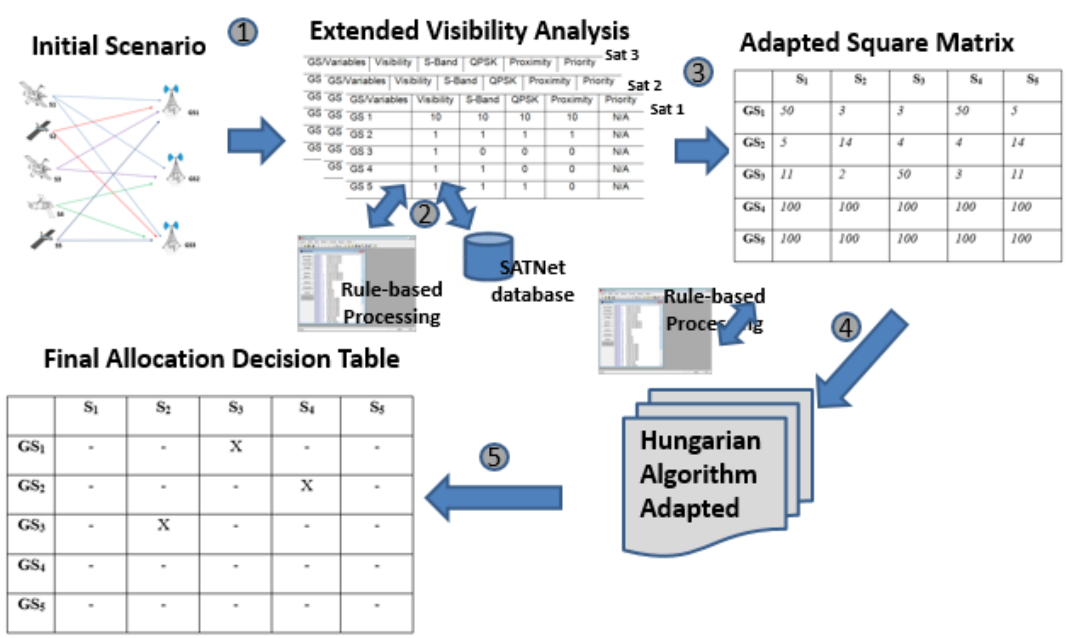
Source: (Espindola, 2017)

2.2. Autonomous agent

The creation of an autonomous agent allows for the solving of various problems. (Arens, 1994) Take an initial scenario; in this case, three stations and five satellites are used. Each satellite has several variables, which will be analyzed in order to make decisions when allocating resources in an optimal manner.

Figure 2 describes the behavior of the ADSGS agent in a scenario where there are 3 satellites and 5 stations, the agent must determine the optimal way of allocating resources, these are matching a satellite to a station.

Figure 2
ADSGS autonomous agent



Source: (Espindola, 2017)

In order to achieve the optimal allocation, ADSGS agent will follow the next five steps:

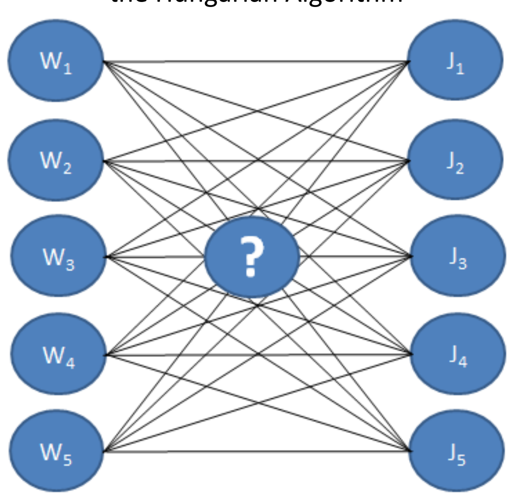
1. Initial scenario features three satellites in Low Earth Orbit (LEO), orbit and stations located around the Earth.
2. Each satellite has its own characteristics defined in the Flight Operational Plan supplied by the SATNet database which in turn, is evaluated by the rule-based expert system created for this purpose. Some of these variables are: visibility from each station, whether it requires S-band for telemetry, type of communication scheme (e.g. QPSK), proximity from the satellite to the ground station and the priority it has.
3. As a result of this analysis, a matrix. This case provides a 3x5 matrix, but running the AHA requires a square matrix, so in order to convert it into a square one an adaptation is made by adding high values (100 for example) in the values that will never be reached or that have no values, such as the absence of ground stations 4 and 5. To create this new matrix the agent is based on previous decisions made using the rule-based expert system.
4. The Hungarian Algorithm, created in 1955 (Kuhn, 1955), in this thesis it is proposed to make a modification, the adaptation of this algorithm will allow the development of the ADSGS agent part of this proposal. The AHA, is executed. This algorithm optimizes the resources allocation and allows the ADSGS agent to make the best decision.
5. Finally, a table is obtained in which a satellite is assigned to each ground station, complying with the considerations and requirements of the proposed scenario.

2.3. Hungarian algorithm

The Hungarian method, created by Hungarian mathematician Harold W. Kuhn, is formally a multi-step procedure for applying its theorem given a cost-matrix of order n and obtaining another matrix with nonnegative entries containing an assignment consisting entirely of zeros. (Kuhn, 1955)

A classical problem solved by the Hungarian algorithm is the one when it is necessary to assign many jobs to many workers where it is required to optimize the minimum time to accomplish the job. Figure 3 shows the depicted situation. The solution of the problem consists of optimizing the time so that only one worker can do one job, but which one?

Figure 3
A classical problem solved by
the Hungarian Algorithm



Source: (Espindola, 2017)

This problem uses a Hungarian Algorithm where the following theorem is proposed: if a number is added to or subtracted from all entries in a row or column of a cost matrix, then an optimal assignment of tasks to the resulting cost-matrix is also an optimal task assignment for the original cost matrix (which is built from supply variables).

The target function to be optimized, according to the structure of the Hungarian method, is given by the Eq. 1 and Eq. 2:

$$Optimize\ \ \boldsymbol{CX} \qquad (1)$$

$$Under\ the\ restriction \quad \boldsymbol{AX = 1} \qquad (2)$$

Where:

C, corresponds to an n×n matrix, of costs generated or caused when job W_k is carried out by worker J_h.

X, is the matrix of variables n×n, the components of which may take the value one or zero, whether the job W_k is assigned or not to worker J_h.

A, is a unitary matrix n×n, with value 1 in each of its components.

1, is an n×1 vector, the components of which are 1.

2.4. Problem formulation

Problematic situations that take place throughout the allocation of a satellite (job) which is orbiting in the space of a ground station (worker) take into account several aspects:

- There are many satellites, all of them different, with different missions and devices that present different characteristics.

- There are many ground stations located in different parts in the world, each one with different devices and characteristics.

- There are more satellites (n) than ground stations (m), thus m < n. According to the latest data by NASA (Satre, 2019) and Online Satellite Calculations (NASA, 2019), at present, there are close to 3500 artificial satellites working and approximately 1000 ground stations.

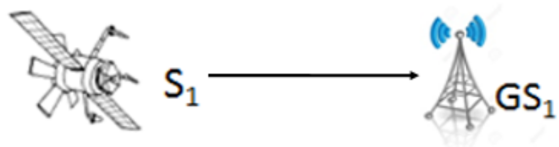
- There is no control over the exact time that a satellite passes over a station, although we do know the exact time, the minute and even the second in which the satellite is going to pass by the ground station.

- A ground station can only attend to a single satellite at a given time. In a short period, it has to be decided which satellite is going to be attended to, even more so when two or more satellites pass over a ground station simultaneously.

2.5. Possible situations

1. When there is only one satellite (S) and only one ground station (GS), n = m = 1. In this case, there is no problem: the ground station is allocated to that satellite. (See Fig. 4)

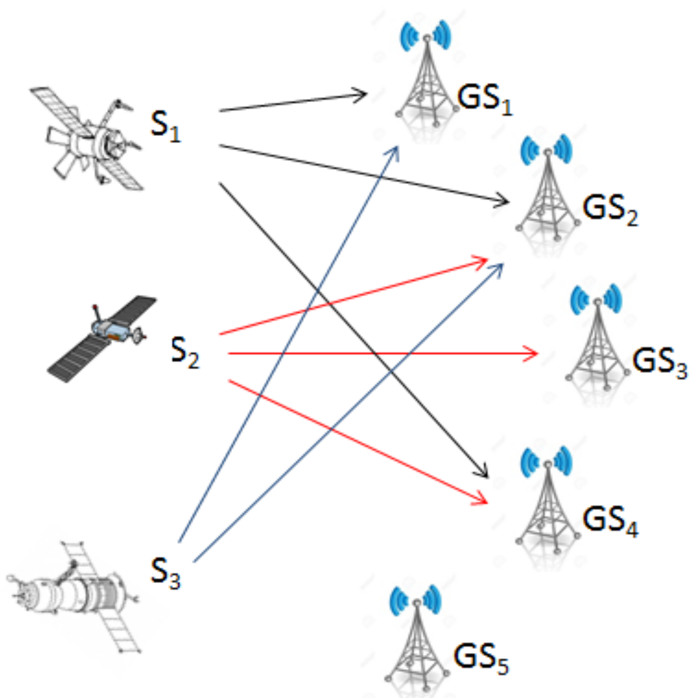
Figure 4
n=1 satellite, m=1 ground station



Source: (Espindola, 2017)

2. When there are n satellites and m ground stations with m ≥ n. (See Fig. 5). In this case, there is no problem as the number of ground stations is higher than the number of satellites, ensuring that each satellite will be attended to.

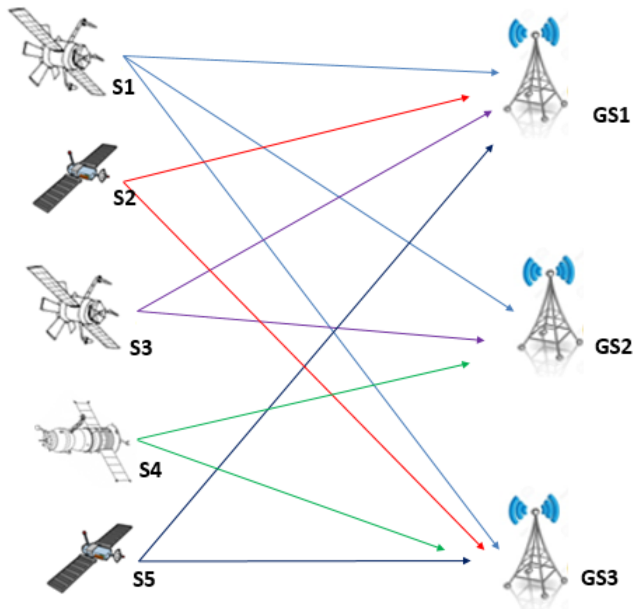
Figure 5
n=3 satellites, m=5 ground stations



Source: (Espindola, 2017)

3. Another situation is presented when there are n satellites and m ground stations with m ≤ n. (See Fig. 6). For example, satellite 1 (S1) can be allocated to any of the three stations (GS), whereas satellite 2 (S2) can only be allocated to either station GS1 or GS3.

Figure 6
n=5 satellites, m=3 ground stations

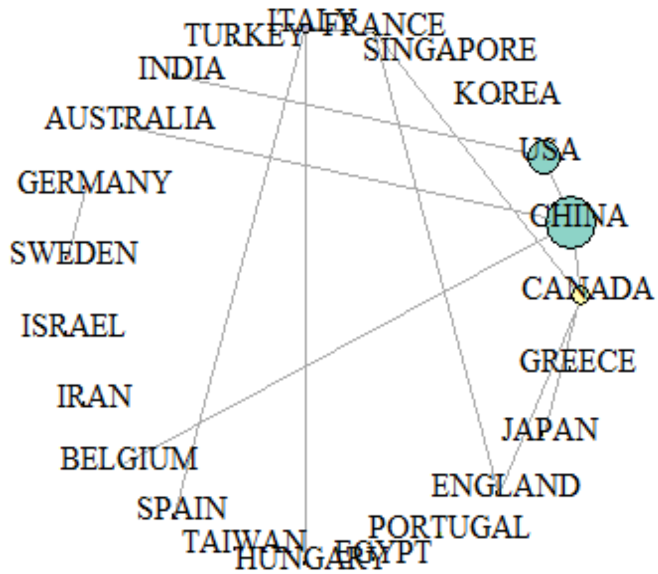


Source: (Espindola, 2017)

In the previous cases, the most complex situation, when it comes to allocation problems, is case 3, given that some satellites would not be attended to when passing over the ground station. This is because the station can only attend to one satellite at a time. Therefore, this research work approaches this case of greater complexity, presenting a solution.

Although there are various optimization methods to solve (1), a recurrent one is the Hungarian method, used to solve transport allocation problems due to the high number of variables and elements that can be included in the problem (A. Ramos, 2010) (Revenga, 2008). This method continues to be cited in different publications in the academic and research world, as can be seen in Fig. 7. It shows the existing scientific collaboration among countries regarding the topic. This was done taking as a basis the study of 48 articles published in journals visible in Web of Science -WoS in the period 2009 to 2018. (Rodríguez, Cely, & Gómez, 2018)

Figure 7
Collaboration among countries. WoS Publications Hungarian Method



Source: (Rodríguez, Cely, & Gómez, 2018)

In Fig. 7, the collaboration can be seen through a network structure with nodes and connection lines. The network structure implies that the greater the diameter of the circumference, the higher the number of publications in the country; and the thicker the line, the stronger the alliance due to the frequency of articles published (Jimenez, Gómez, & Rodríguez, 2019). Network structure applied in other areas, showing new dynamics of cooperation in the production of scientific knowledge (Gómez, Soto, & Lima, 2018) y (Rodríguez, Gómez, & Herrera, 2017).

3. Results

Next, the mathematical model, the input variables, the cost matrix, and finally the representation of the solution for the resource allocation problem are shown. In this case it allows us to define which satellite will be served by the ground station through which it is passing at that instant of time.

3.1. Mathematical model

The optimization process is carried out through the Hungarian method, which needs to find a minimum associated cost; in this case, to the fact that ground station GS_h receives or attends to a satellite S_k. The algorithm requieres n ground stations and n satellites, with the aim of minimizing Eq. 1, that is to say the Eq. 3:

$$Minimize\ \boldsymbol{CX} \qquad (3)$$

where

C, corresponds to an n×n matrix of costs generated or caused when attending to satellite S_k, in the ground station GS_h.

X is the n×n matrix of variables, the components of which can take a value one or zero, whether satellite k is allocated to ground station h or not.

The Eq. 3 has restrictions according to Eq. 2.

3.2. Supply Variables

Given a ground station and in order to decide what satellite to attend to, the variables that define the problem have to be taken into consideration. Some are variables of the satellite; others are of the ground station. With the aim of weighing up the allocation, certain criteria are established, which generate a matrix of supply variables (V).

Table 1 contains the supply variables: considering 1 satellite, h ground stations and p supply variables.

Table 1
Supply variables, for *m* ground stations, 1 satellite

	Satellite 1 (S_1)				
GS\Variables	V_1	V_2	…	V_P	TOTAL
GS_1	v_{111}	v_{112}	…	v_{11p}	c_{11}
GS_2	v_{211}	v_{212}	…	v_{21p}	c_{21}
⋮	⋮	⋮	⋱	⋮	⋮
GS_h	v_{h11}	v_{h12}	…	v_{h1p}	c_{h1}

Where vj1i, corresponds to the value (in this case it represents a cost) which takes the supply variable i-ésima, in the ground station j-ésima, for the satellite S_1. The value cj1=$\sum_{i=1}^{p} v_{j1i}$ corresponds to the sum of the values of the supply variables in the j-ésima ground station, for satellite S_1.

3.3. Costs Matrix

The table of costs is built from the table of supplies, where each cell (*h*, *k*) is obtained from the sum of the values that the supply variables take ($v_h k_i$): h represents a ground station, *k* a satellite, and *i* a supply variable. The table of costs is presented in Table 2, in which the first column corresponds to the column headed TOTAL in/of Table 1 with *m*=*h*.

Table 2
Costs for *m* ground stations, *n* satellites and *p* variables

	S_1	S_2	…	S_n
GS_1	$c_{11}=\sum_{i=1}^{p} v_{11i}$	$c_{12}=\sum_{i=1}^{p} v_{12i}$	…	$c_{1n}=\sum_{i=1}^{p} v_{1ni}$
GS_2	$c_{21}=\sum_{i=1}^{p} v_{21i}$	$c_{22}=\sum_{i=1}^{p} v_{22i}$	…	$c_{2n}=\sum_{i=1}^{p} v_{2ni}$
⋮	⋮	⋮	⋱	⋮
GS_m	$c_{m1}=\sum_{i=1}^{p} v_{m1i}$	$c_{m2}=\sum_{i=1}^{p} v_{m2i}$	…	$c_{mn}=\sum_{i=1}^{p} v_{mni}$

The previous table of costs is of an m×n type, being m < n. In order to implement the algorithm with the Hungarian method, it is necessary to build a square table of costs, of the type corresponding to the maximum between the number of stations and the number of satellites, corresponding AHA process, with a table of n×n type.

According to case 3, mentioned in the previous section, given that there are more satellites (S) than ground stations (GS), it will be necessary to add fictitious ground stations, such as GS_{m+1}, GS_{m+2}, …, GS_n and are called dummy ground stations. Thus, Table 2 acquires the structure of Table 3 (Table of costs).

Table 3
Table of costs, including dummy ground stations *n-m*

	S₁	**S₂**	**…**	**Sₙ**
GS_1	$c_{11}=\sum_{i=1}^{p} v_{11i}$	$c_{12}=\sum_{i=1}^{p} v_{12i}$	…	$c_{1n}=\sum_{i=1}^{p} v_{1ni}$
GS_2	$c_{21}=\sum_{i=1}^{p} v_{21i}$	$c_{22}=\sum_{i=1}^{p} v_{22i}$	…	$c_{2n}=\sum_{i=1}^{p} v_{2ni}$
⋮	⋮	⋮	⋱	⋮
GS_m	$c_{m1}=\sum_{i=1}^{p} v_{m1i}$	$c_{m2}=\sum_{i=1}^{p} v_{m2i}$	…	$c_{mn}=\sum_{i=1}^{p} v_{mni}$
GS_{m+1}	$c_{m+1,1}=\sum_{i=1}^{p} v_{(m+1)1i}$	$c_{m+1,2}=\sum_{i=1}^{p} v_{(m+1)2i}$	…	$c_{m+1,n}=\sum_{i=1}^{p} v_{(m+1)ni}$
⋮	⋮	⋮	⋱	⋮
GS_n	$c_{n1}=\sum_{i=1}^{p} v_{n1i}$	$c_{n2}=\sum_{i=1}^{p} v_{n2i}$	…	$c_{nn}=\sum_{i=1}^{p} v_{nni}$

In order to implement the Hungarian method, the cells (h, k) of the dummy variables (being h = m+1, m+2, …, n and k = 1, 2, .., n); the cost c_{hk} assigned to each cell of the table of costs, will take the following value (Eq. 4)

$$hk = \sum_{i=1}^{p} 2\omega = 2\omega * p. \qquad (4)$$

Thus, in Table 3 costs will have to be built (n-m) ×n cells associated with dummy ground stations, with value 2ω * p. Two is an arbitrary value and it is possible to use a higher number.

Where ω, is a "high" value in comparison to the costs allocated to the cells of the non-dummy ground stations; the value of ω can also be used for cases in which the conditions established in the variables do not apply or no value can be established. This makes that the cells where the value of ω is allocated, by the Hungarian algorithm, have a lower priority to conform the optimal solution to the problem (3).

Table 3 generates the costs matrix C of $n \times n$ type, defined in the allocation problem of the Eq. 3; that is to say

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \vdots & c_{nn} \end{bmatrix}$$

Taking into account the Eq. 3, the allocation problem has n×n variables, which are represented by x_{hk} and form the matrix X, where h represents the ground station and k represents the satellite. For the case of the Hungarian method see Eq. 5.

$$x_{hk} = \begin{cases} 1 & \text{if the ground station } \mathrm{GS}_h \text{ is assigned to satellite } \mathrm{S}_k \\ 0 & \text{if the ground station } \mathrm{GS}_h \text{ is not assigned to satellite } \mathrm{S}_k \end{cases} \qquad (5)$$

Table 4 presents a structure of the table of variables of the allocation problem posed in the Eq. 3. These variables can take values 1 or 0.

Table 4
Table of variables of the allocation problem

	S_1	S_2	...	S_n
GS_1	x_{11}	x_{12}	...	x_{1n}
GS_2	x_{21}	x_{22}	...	x_{2n}
⋮	⋮	⋮	⋱	⋮
GS_n	x_{n1}	x_{n2}	...	x_{nn}

In the allocation problem, Table 4 allows for building the permutation matrix, which is a binary matrix of $n \times n$ type, symbolized by

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \vdots & x_{nn} \end{bmatrix}$$

According to the Eq. 3, the function that has to be minimized is the following:

$$\min \sum_{h=1}^{n} \sum_{k=1}^{n} c_{hk} x_{hk}$$

Which is conditioned (Eq. 2) by the following restrictions:

$$\sum_{k=1}^{n} x_{1k} = 1, \quad \sum_{k=1}^{n} x_{2k} = 1, \quad \ldots \quad \sum_{k=1}^{n} x_{nk} = 1,$$
$$\sum_{h=1}^{n} x_{h1} = 1, \quad \sum_{h=1}^{n} x_{h2} = 1, \quad \ldots \quad \sum_{h=1}^{n} x_{hn} = 1.$$

The solution to the previous allocation problem consists of establishing exactly 1 in some of the entries (cells) of each row and allocating exactly 1 to any of the entries (cells) of each column of the X matrix; that is to say, allocating a sole ground station to a single satellite.

3.4. Representation of the Solution

For the case of Fig. 6, where three ground stations are involved (m=3) and five satellites (n=5), a supply matrix is created, with five variables (p=5:VS, FR, MO, PX, PR), described below:

- V1: Visibility (VS): from a ground station, visibility is 0 when it is 100% visible and 1 when it is definitely not visible.

- V2: Frequency (FR): this is the signal in which the satellite will transmit or receive information from the station. This variable takes value 0 if there is compatibility of the type of frequency between the satellite and the ground station. For example, if the frequency is Ultra High Frequency (UHF), then FR=0, otherwise FR=1.

- V3: Modulation (MO): the modulation scheme is formed by the way which information is received, whether it is from the ground station or from the satellite. This variable takes a 0 value if there is compatibility of the modulation type between the satellite and the ground station. For example, if the modulation is QPSK then MO=0, otherwise MO=1.
- V4: Proximity (PX): how close is the satellite from the ground station? If the signal (communication, telemetry or telecommand) between the satellite and the station is reachable and with less distance than to other satellites, then PX=0, otherwise PX=1.
- V5: Priority (PR): when the owner of the satellite urgently requires to carry out a telemetry (receive information from the satellite) or a telecommand (send information to the satellite), they can change the priority. PR=0 if it is a maximum priority PR=1 if it is not a priority at the time.

The number of supply variables (V_i) in a problem could be higher or lower than the one defined here. In addition, they can be of a different type and according to a fixed scale.

In Table 5, the case of a satellite, 3 ground stations and 5 supply variables is exemplified.

Table 5
Table of variables of satellite S_1

GS\Variables	**Visibility ($VS=V_1$)**	**Frequency ($FR=V_2$)**	**Modulation ($MO=V_3$)**	**Proximity ($PX=V_4$)**	**Priority ($PR=V_5$)**	**TOTAL**
GS_1	10	10	10	10	10	$c_{11}=50$
GS_2	1	1	1	1	1	$c_{21}=5$
GS_3	1	0	0	0	10	$c_{31}=11$

The supply variables can take the $\omega = 10$ value or a higher value (see Eq. 4), when the conditions established in the description of the variables do not apply (0 or 1, depending on what was previously established), or it is not possible to establish a value for the variable. In the previous table, the values of the column headed TOTAL are the result of:

Table 6
Table of costs for satellite S_1

Station/Satellite	S_1
GS_1	$c_{11}=VS_{11}+FR_{11}+MO_{11}+PX_{11}+PR_{11}$
GS_2	$c_{21}=VS_{21}+FR_{21}+MO_{21}+PX_{21}+PR_{21}$
GS_3	$c_{31}=VS_{31}+FR_{31}+MO_{31}+PX_{31}+PR_{31}$

The costs of the supply variables for the four satellites are indicated in Table 7.

Table 7
Table of variables: satellite supplies 2 to 5

GS	**V_1**	**V_2**	**V_3**	**V_4**	**V_5**	**TOTAL**
GS_1	0	1	1	1	0	$c_{12}=3$
GS_2	1	1	1	10	1	$c_{22}=14$
GS_3	1	0	0	0	1	$c_{32}=2$

Satellite2

GS	**V_1**	**V_2**	**$_3$**	**V_4**	**V_5**	**TOTAL**
GS_1	0	0	1	1	1	$c_{13}=3$
GS_2	1	1	1	1	0	$c_{23}=4$
GS_3	10	10	10	10	10	$c_{33}=50$

Satellite3

GS\	V_1	V_2	V_3	V_4	V_5	TOTAL
GS_1	10	10	10	10	10	c_{14}=50
GS_2	1	1	1	1	0	c_{24}=4
GS_3	1	1	0	0	1	c_{34}=3

Satellite4

GS\	V_1	V_2	V_3	V_4	V_5	TOTAL
GS_1	1	1	1	1	1	c_{15}=5
GS_2	1	1	1	1	10	c_{25}=14
GS_3	1	0	0	0	10	c_{35}=11

Satellite5

The associated costs (Table 8) are generated by replacing the values in Tables 5 and 7.

Table 8
Table of costs

	S_1	S_2	S_3	S_4	S_5
GS_1	$c_{11}=50$	$c_{12}=3$	$c_{13}=3$	$c_{14}=50$	$c_{15}=5$
GS_2	$c_{21}=5$	$c_{22}=14$	$c_{23}=4$	$c_{24}=4$	$c_{25}=14$
GS_3	$c_{31}=11$	$c_{32}=2$	$c_{33}=50$	$c_{34}=3$	$c_{35}=11$

Based on the previous table, the costs matrix is generated, which has the following form:

$$C = \begin{bmatrix} 50 & 3 & 3 & 50 & 5 \\ 5 & 14 & 4 & 4 & 14 \\ 11 & 2 & 50 & 3 & 11 \end{bmatrix}$$

Table 9
Values: Table of costs before using the Hungarian algorithm

	S_1	S_2	S_3	S_4	S_5
GS_1	50	3	3	50	5
GS_2	5	14	4	4	14
GS_3	11	2	50	3	11
GS_4	100	100	100	100	100
GS_5	100	100	100	100	100

After using the Hungarian algorithm, Table 10 is obtained, which shows the results of the allocation of satellites to the ground stations.

Table 10
Allocation results

	S_1	S_2	S_3	S_4	S_5
GS_1	-	-	X	-	-
GS_2	-	-	-	X	-
GS_3	-	X	-	-	-
GS_4	-	-	-	-	-
GS_5	-	-	-	-	-

This means that the implementation of AHA for this optimization problem, which corresponds to the allocation of satellites to stations can be solved satisfactorily. Illustrating the case with five satellites and three stations, the result is: satellite 2 will be served by station 3, satellite 3 by station 1, and satellite 4 by ground station 2, within a certain time (t).

Optimize the use of different resources (software and hardware) used in ground station networks for satellite tracking and control, where definitely the amount of satellites, it will always be greater than that of ground stations, it allows to increase the reliability in the success of the mission. Similarly, it increases the development of this area of space technology, since it considerably reduces the operating costs of the space system.

The use of a mathematical model strengthens and validates the research carried out to optimize the use of earth station networks used in satellite tracking and control.

4. Conclusions

The formalization of AHA's optimization structure to problems associated with the allocation of resources facilitates its application and expands the spectrum of situations to solve beyond what is represented by the issue with satellites and ground stations.

The adaptation of the Hungarian Algorithm makes a contribution in the solution of problems of allocation since it allows the conversion of non-square matrices into square matrices, thus, adapting to the solution of more problems of this type.

The proposed mathematical model, solve the problem of resource allocation in ground station networks and contributes to the development of space technology.

It is expected that this model will be used in the development of the ground segment, and contribute to satellite tracking and control from ground stations and optimize this process.

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